Toys

First 100: Dawid Jamka, Poland (14:31)
#AC = 26

problem author: Karol Pokorski

Triangles

First 100: Mariusz Trela, Poland (58:17)
#AC = 18

problem author: Kamil Dębowski

Fibonacci

Max score: 65
First 65: Costin-Andrei Oncescu, Romania (2:19:09)
#65 = 2

problem authors: Dominik Klemba, Kamil Dębowski

Fibonacci

$0100101 = F_2 + F_5 + F_7 = 2 + 8 + 21 = 31$

1) Let's find $X(F_{a[1]})$

0000001
0000110
0011010
1101010

$X(F_{a[1]}) = (a[1]-1)/2$

Fibonacci

2) Let’s find $X(F_{a[1]}+F_{a[2]})$

00000001000001
00000001000110
00000001011010
00000002101010
00000111101010
00011011101010
…

$X(F_{a[1]}+F_{a[2]}) \approx (a_1 / 2) \cdot ((a_2-a_1) / 2)$

Fibonacci

3) General case, values a_i far away from each other.

$X \approx (a_1 / 2) \cdot ((a_2-a_1) / 2) \cdot ((a_3-a_2) / 2) \cdot \ldots$

$X \approx (d_1 / 2) \cdot (d_2 / 2) \cdot (d_3 / 2) \cdot (d_4 / 2) \cdot \ldots$

(d_i are distances between sorted a_i)

Fibonacci

The $O(n^2)$ solution, $|a_i - a_j| \geq 2$

For every prefix, sort values $a_1, a_2, \ldots, a_k$, and run O(k) dynamic programming dp[2].

…00010001…
…00010110…
…00021010… ← dp[0] is the number of ways to choose values on the right so that the next 1 on the left must be changed to smaller 1's (pushed further to the left)

dp[1] means: we can leave the next 1 unchanged

Fibonacci

…00010001…
…00010110…
…00021010… ← dp[0] is the number of ways to choose values on the right so that the next 1 on the left must be changed to smaller 1's (pushed further to the left)
dp[1] means: we can leave the next 1 unchanged

dp'[0] = dp[0] + dp[1] (if distance is even, else 0)

dp'[1] = (dp[0] + dp[1]) · (distance - 1) / 2 + dp[1]

Fibonacci

What if $a_j = a_i + 1$?

…001100…
…000010…

Possible chain effect, amortized O(n) in total

…00010101010…
…00**1**10101010…
…0000**1**101010…
…000000**1**1010…
…

Fibonacci

“a_i are different squares of natural numbers”

“a_i are different even numbers”

No collisions.

Then we already have an $O(n^2)$ solution.

Fibonacci

What if $a_j = a_i$?

…000200…
…010010… because $2 \cdot F_k = F_{k+1} + F_{k-2}$

Doesn’t amortize :(

Fibonacci

What if $a_j = a_i$?

Doesn't amortize :(

…000101010**1**00…
…000101010**2**00…
…000101020110…
…000102011110…
…000201111110…
…010111111110…
…010001010101…

Fibonacci

$O(n^2)$ solution: resolve conflicts in any way in recompute the answer in O(n) each time

Let’s try to avoid recomputing the answer.

Fibonacci

Let’s try to avoid recomputing the answer.

- a segment tree (either off-line or BST)

- matrix 2x2 or 3x3 in every node

- O(log(n)) per change

Fibonacci

Last steps.

way I – distances between consecutive 1’s

way II – maximal intervals of type 1010101

Thank you for your attention.

Good luck on IOI!